DESCRIPTION OF TOPICS AND SUBTOPICS

Math4Student and Math4Teacher Applications

The educational portal math4u.vsb.cz comprises three main sections – Math for Student, Math for Teacher and Math for Class.

In the Math for Student (STUDENT) you can generate an interactive HTML test from the database of 4,000 questions. The questions are sorted into twelve topics and further divided into 56 smaller subtopics for practicing all areas of secondary school mathematics. In the section Math for Teacher (TEACHER) everybody can prepare a tailor-made interactive test or a printer ready version of a written test. You can choose any questions you like from the same database of 4,000 questions as students use in the application Math for Student. The section Math for Class contains 152 interesting quizzes and educational games so called “matching games” and “table-selection games”.

All tests and educational games are available in five languages – English, Czech, Slovak, Polish and Spanish. And everything is free of charge!

Basics of Math

Logic and Sets

Part	Description
A	• Basic set operations (intersection, union, set difference, complement) • Sets specified by a characteristic property
B	• Math logic statements, truth values of statements, quantifiers • Set operations – complex problems
C	• Word problems – solvable with aid of Venn diagrams

Elementary Arithmetics

Part	Description
A	• Calculations with fractions and decimals • Rounding • Writing numbers in exponential form
B	• Numbers divisibility

Polynomials and Fractions

Part	Description
A	• Basic operations with polynomials (addition, subtraction, multiplication, use of formulas for $(a+b)^2 and\ (a-b)^2$ • Simplifying algebraic expressions • Evaluating algebraic expressions
B	• Factoring polynomials into products • Simplifying algebraic expressions – complex problems • Problems solvable with use of formulas for $(a+b)^3 and\ (a-b)^3$ • Finding of all values such that an expression is not defined • Finding of all values such that an expression equals zero • Word problems – isolating the variable out of a formula
C	• Division of two polynomials • Problems solved using Binomial theorem • Problems solved using formulas for $a^3+b^3 and\ a^3-b^3, \ldots$

Expressions with Powers and Roots

Part	Description
A	• Powers with natural exponents • Second and third root • Simplifying fractions with roots in denominator
B	• Powers with integer or rational exponents • Higher roots • Comparing values of expressions
C	• Simplifying expressions with powers and roots – complex problems • Simplifying fractions with roots in denominator – complex problems • Comparing values of expressions - complex problems

Absolute Value

Part	Description
A	• Evaluating absolute value of numerical expressions
B	• Geometric interpretation of the absolute value • Simplifying expressions with absolute value • Simple equations and inequalities with absolute value
C	• Properties of the absolute value

Percent Problems

Part	Description
A	• Calculation of percentages – standard problems
B	• Calculation of percentages and percentage points – complex problems
C	• Calculation of percentages – complex word problems (price growth, inflation, interests) • Problems leading to systems of equations

Equations and Inequalities

Linear Equations and Inequalities

Part	Description
A	• Simple linear equations • Equivalent equations • Graphical solutions of linear equations • Linear equations specified by word description
B	• Simple linear inequalities • Graphical solutions of linear inequalities • Linear inequalities specified by word description
C	• Word problems leading to linear equations and inequalities

Quadratic Equations and Inequalities

Part	Description
A	• Quadratic equations
B	• Quadratic inequalities • Vieta's formulas • Word problems leading to quadratic equations and inequalities
C	• Quadratic equations and inequalities with absolute value • Word problems – more complex

Higher Degree Equations and Inequalities

Part	Description
A	• Equations solvable by factoring polynomials to products of linear and quadratic factors
B	• Equations solvable by substitution method • Cubic equations with one of the roots known • Inequalities solvable by factoring polynomials to products of linear and quadratic factors
C	• Equations of 4^{th} degree with two of the roots known • Equations of higher degrees, to guess some of the roots is necessary

Systems of Equations and Inequalities

Part	Description
A	• Systems of two linear equations with two unknowns • Graphical solutions of systems of two linear equations with two unknowns • Single equation with two unknowns
B	• Systems of two linear inequalities with two unknowns • Graphical solutions of systems of two linear inequalities • One inequality with two unknowns • Word problems leading to systems of two linear equations or inequalities with two unknowns
C	• Systems of two linear equations with two unknowns and a parameter • Systems of three linear equations with three unknowns • Matrix calculus – matrix, the rank of a matrix • More complex systems of equations or inequalities (quadratic, with absolute value, with rational expressions, ….) • Word problems

Rational Equations and Inequalities

Part	Description
A	• Rational equations • Domains of rational equations
B	• Rational inequalities • Domains of rational inequalities
C	• Rational equations and inequalities with absolute values

Absolute Values Equations and Inequalities

Part	Description
A	• Linear equations and inequalities with single absolute value - solutions based on geometric interpretation of the absolute value
B	• Linear equations with one or more absolute values
C	• Linear inequalities with one or more absolute values • Linear equations and inequalities with absolute values inserted in absolute values

Radical Equations and Inequalities

Part	Description
A	• Equations with an unknown under a single radical • Domain of an equation • Domain of an expression with an unknown under a radical
B	• Equations with unknown under several radicals • Inequalities with unknowns under radicals
C	• Word problems • More complex equations – combinations of radicals and absolute values

Equations and Inequalities with Parameters

Part	Description
A	• Linear equations with a parameter • Equations and inequalities with a parameter solved for a given value of the parameter
B	• Linear inequalities with a parameter • Quadratic equations and inequalities with a parameter • Rational equations and inequalities with a parameter

Functions

Properties of Functions

In this subarea problems to practice properties of functions are included. To solve problems in part A precise knowledge of individual functions is not required, since functions are given by tables or graphs.

Problems included in part B are convenient to practice properties of concrete functions as quadratic, rational, power functions and functions with absolute values. Also, there are combined problems with different types of functions and with composite functions in part B.

Part C contains problems to practice the notions of one-to-one function and inverse function through other various types of functions.

Part	Description
A	• Properties of functions given by a table or a graph (parity, monotonicity, minima, maxima)
B	• Properties of functions given by equations – practicing through various types of functions (linear, quadratic, with absolute values, rational)

	• Domains of composite functions
C	• One-to-one function and inverse function

Linear Functions

Part	Description
A	• Properties of linear functions and of their restrictions (domain, range, monotonicity, intercepts with axis, …) • Function values • Equation of a linear function • Verifying if the given point lies on a graph of a function
B	• Transformations of a graph of a linear function • Using graphs of functions to find all values of x such that $f(x)<g(x)$
C	• Finding equation of a linear function (complex problems) • Linear functions with parameter • Word problems

Quadratic Functions

Part	Description
A	• Properties of quadratic functions (domain, range, intercepts with axis, monotonicity, …) • Determining of function values • Matching graphs to equations of corresponding functions
B	• Transformations of a graph of a quadratic function • Determining of the equation of a function given by three points • Determining of the vertex of a parabola • Solving quadratic equations and inequalities with an aid of graphs of quadratic functions
C	• Quadratic functions with parameter • Quadratic functions with absolute values • Solving quadratic equations and inequalities with absolute value with an aid of graphs of quadratic functions • Word problems

Functions with Absolute Values

All problems included in this subarea are on linear functions with absolute values. Problems on functions with absolute values that are not linear are to be found in sections related to specified types of functions. I.e., quadratic functions with absolute values are to be found in the section on quadratic functions.

Part	Description
A	• Properties of absolute value function • Function $f(x) = a\|x - b\| + c$, where $a, b, c \in \mathbb{R}$
B	• Functions with absolute values and their graphs • Properties of functions with absolute value (domain, range, monotonicity, extremes, boundedness, parity)
C	• Function with an absolute value inside an absolute value

Power and Radical Functions

Part	Description

A	• Power functions with integer exponent
	  ○ Determining function value
	  ○ Graphs of functions and their transformations
	  ○ Properties of functions (domain, range, monotonicity, extremes, boundedness, parity, …)
	  ○ Inequalities assessments with an aid of graphs of functions

B	• Nth-root function
C	• Functions with absolute values • Word problems

Rational Functions

Part	Description

A	• Inverse proportionality
	  ○ Graph of the function
	  ○ Function value
	  ○ Word problems

B	• Linear rational function
	  ○ Function's graph and its transformations
	  ○ Center of a hyperbola
	  ○ Properties of functions (domain, range, monotonicity, extremes, boundedness, parity)

C	• Rational function • Functions with absolute values

	• Problems with parameters • Word problems

Exponents and Logarithms

Exponential Functions

Part	Description
A	• Definition of the exponential function • Graph and its transformations • Domain and range
B	• Exponential function properties – monotonicity, boundedness • Comparing of function values (with aid of graphs or monotonicity)
C	• Composite functions (with absolute values or radicals) • Practical word problems

Logarithmic Functions

Part	Description
A	• Logarithms and basic counting rules • Definition of the logarithmic function • Graph and its transformations • Domain and range
B	• Logarithmic function properties – monotonicity, boundedness • Comparing of function values (with aid of graphs or monotonicity) • Logarithms counting rules (more complex problems) • Domains of logarithmic expressions
C	• Composite functions (with absolute values or radicals) • Simplifying expressions with logarithms of various base • Practical word problems

Exponential Equations and Inequalities

Part	Description
A	• Equations with the same base - solvable by comparing exponents
B	• Equations with the same base (more complex) - solvable by comparing exponents • Equations solvable by substitution

C	• Inequalities solvable by comparing exponents • Inequalities solvable by substitution • System of inequalities

Logarithmic Equations and Inequalities

Part	Description
A	• Equations with the logarithms of the same base - solvable by comparing arguments • Equations with the logarithms of the same base – solvable with use of logarithm counting rules
B	• Equations with the logarithms of the same base (more complex) – solvable with use of logarithm counting rules • Equations with logarithms of various bases • Equations solvable by substitution • Equations solvable by taking logarithm • System of equations
C	• Inequalities solvable by simplifying and comparing arguments • Inequalities solvable by substitution

Trigonometry

Angles, Arcs and Sectors

Part	Description
A	• Conversions of degrees to radians and vice versa • Coterminal angles, coterminal angles between 0 and 360 degrees. • Correspondence between angles and quadrants • Adding and subtracting angles
B	• Angles specified by given conditions – arithmetic mean, enumeration, … • Computational problems involving clocks, calculation of marching angle (azimuth) • Coterminal angles – complex problems

Sine, Cosine, Tangent and Cotangent

Part	Description
A	• Trigonometric ratios of standard angles

B	• Properties of trigonometric functions – parity, periodicity, boundedness • Domains and ranges • Graphs of trigonometric functions • Sine and cosine relationships
C	• Simplifying expressions with trigonometric functions - use of trigonometric identities • Domains of trigonometric expressions • Trigonometric functions with absolute value

Trigonometric Equations and Inequalities

Part	Description
A	• Basic trigonometric equations • Using substitution for solving trigonometric equations • Using basic identities for solving trigonometric equations
B	• Basic trigonometric inequalities
C	• More complex trigonometric equations and inequalities (use of trigonometric identities, exponentiation, …) • Trigonometric equations and inequalities with absolute value

Triangle Trigonometry

Triangles

Part	Description
A	• Computation of angle measures in a triangle where the angles satisfy the given condition • Relationships between sides and angles of a triangle • Properties of triangles, computational problems
B	• Trigonometric functions in a right triangle • Application problems solved using trigonometric functions • Law of sines and law of cosines
C	• More complex application problems

Polygons

Part	Description

A	• Computational problems on angles, lengths and areas
	  ○ Square
	  ○ Rectangle
	  ○ Rhombus

B	• Computational problems on angles, lengths and areas
	  ○ Trapezoid
	  ○ Parallelogram
	  ○ Regular polygons

C	• Computational problems on angles, lengths and areas
	  ○ Deltoid (Kite)
	  ○ Combined complex problems

Circles

Part	Description
A	• Inscribed and central angle
B	• Angles between tangents • Polygons inscribed to a circle • Disc, annulus • Circular sector and circular segment
C	• Disc, circular sector and circular segment – complex problems

Geometry

Lines and Planes: Intersecting, Perpendicular, Parallel

Part	Description
A	• Point line, half line, line segment, angle - notation • Mutual position of lines in plane • Mutual position of line and circle • Mutual position of two circles
B	• Mutual position of lines in space • Mutual position of line and plane • Mutual position of two (three) lines • Cross-sections of cube and pyramid • Intersections of line with cube and pyramid surfaces

Lines and Planes: Distances and Angles

Part	Description
A	• Word description of angles in a cube • Cube – distances of points, lines, planes • Cube – angles between lines, planes • Cuboid - distances of points, lines, planes • Cuboid – angles between lines, planes
B	• Word description of angles in a pyramid • Square pyramid – distances of points, lines, planes • Square pyramid - angles between lines, planes • Cone – angles
C	• Regular right hexagonal prism - distances and angles • Hexagonal pyramid – distances and angles • Tetrahedron - distances and angles

Volume and Surface Area Formulas

Part	Description

A	• Computation of volumes and surfaces
	  ○ Cube
	  ○ Cuboid

B	• Computation of volumes and surfaces
	  ○ Cone
	  ○ Cylinder
	  ○ Sphere
	  ○ Three or four sided pyramid
	  ○ Right triangular or rectangular prism

C	• Computation of volumes and surfaces
	  ○ Truncated pyramid
	  ○ Truncated cone
	  ○ Regular right hexagonal prism
	  ○ Regular hexagonal pyramid

Symmetry and Geometric Transformations

Part	Description
A	• Point symmetry • Line symmetry (reflection symmetry)

B	• Translation • Rotation
C	• Dilation

Analytic Geometry

Points and Vectors

Part	Description
A	• Points and vectors in plane and in space • Length of a vector • Operations with vectors – sum, scalar multiple • Linear combination of vectors • Linear dependence of vectors • Line segment – center, length • Triangle – centroid, centers of sides, lengths of sides, perimeter
B	• Scalar product (dot product) of vectors in plane and in space • Perpendicular vectors • Parallel vectors • Angle of vectors • Applications – plane shapes, solids in coordinate system
C	• Vector product of vectors • Area of a plane region, area of a face of a solid • Volume of a solid (parallelepiped, pyramid, tetrahedron) • Complex problems covering whole topic

Analytic Geometry in a Plane

Part	Description
A	• Line – parametric description, general equation, point-slope form equation • Direction vector and normal vector of a line • Line segment, half line – parametric description • Relative position of two lines • Perpendicularity of lines • Parallelity of lines
B	• Distance of a point from a line • Distance of two parallel lines

	• Angle of two lines • Triangle – medians, heights (altitudes), side perpendicular bisectors • Line and point reflection, translation
C	• Angles and distances – more complex problems • Complex problems covering whole topic

Analytic Geometry in a Space

Part	Description
A	• Line – parametric description • Half line, line segment – parametric description • Plane - parametric description, general equation • Intersection of two lines • Intersection of a line and a plane • Intersection of two planes • Relative position of points, lines and planes
B	• Perpendicularity of lines and planes • Parallelity of lines and planes • Angles of lines and planes • Distance of a point from a plane • Distance of a point from a line • Intersection of two planes – more complex problems
C	• Point, line and plane reflection • • • Metric problems on solids • Mutual position of three planes • Complex problems on perpendicularity

Conics

Part	Description
A	• Circle (center and radius) • Ellipse (center, semi-major and semi-minor axis, foci, vertex and co-vertex)
B	• Parabola (vertex, directrix, focus) • Hyperbola (center, foci, vertices, major and minor axis, excentricity)
C	• Tangent line to a conic • Conic and a line • Conic passing through given points • Word problems

Complex numbers

Complex Numbers in Algebraic and Trigonometric Form

Part	Description
A	• Imaginary unit • Algebraic form of a complex number – addition, subtraction, multiplication, division • Complex conjugate of a complex number • Geometric representation of complex numbers in gaussian plain • Absolute value of a complex number
B	• Trigonometric form of a complex number – the argument, the absolute value (modulus) • Trigonometric form of a complex number – multiplication, division • Trigonometric and algebraic form conversion of complex numbers
C	• Simple equations of two variables with complex coefficients

De Moivre's Theorem

Part	Description
A	• De Moivre's theorem (powers of complex numbers)

Quadratic Equations With Complex Roots

Part	Description
A	• Quadratic equations with real coefficients • Factoring of quadratic trinomial
B	• Quadratic equations with real coefficients (complex problems) • Quadratic equations with real coefficients with parameter
C	• Quadratic equations with complex coefficients

Binomial Equations

Part	Description
A	• Solving binomial equations

Combinatorics, Probability and Statistics

Combinatorics

Part	Description
A	• Combinatorial product rule and sum rule • Arrangements without repetition / k-permutations without repetition • Arrangements with repetition / k-permutations with repetition • Permutations without repetition • Permutations with repetition • Selections without repetition / k-combinations without repetition
B	• Simplifying expressions with factorials and binomial coefficients • Combinatorial equations
C	• Selections with repetition/ k-combinations with repetition • Combinatorial inequalities • Binomial theorem

Probability

Part	Description
A	• Classical probability definition
B	• Geometrical probability • Probability of complementary event • Probability of union of events • Probability of intersection of independent events
C	• Binomial distribution (Bernoulli scheme) • Conditional probability

Statistics

Part	Description
A	• Measures of location (mean, median, mode) • Arithmetic, geometric and harmonic mean
B	• Measures of variability (variance, standard deviation, coefficient of variation)
C	• Summary statistics • Correlation coefficient

Sequences and Series

Introduction to Sequences

Part	Description
A	• Ways to specify a sequence • Finding of one or more members of a sequence
B	• Defining a sequence (nth term formula or recurrent relation) • Properties of sequences (strictly increasing or strictly decreasing, nonincreasing, nondecreasing, upper or lower bounded, bounded) • Finding the nth term of a sequence

Arithmetic Sequences

Part	Description
A	• Defining a sequence (nth term formula or recurrent relation) • Finding the nth term of a sequence • Finding the common difference of a sequence
B	• Sum of the first n terms of a sequence • Sequences of non-numerical terms • Systems of equations containing sequence terms
C	• Word problems • Equations and inequalities containing sums of sequences

Geometric Sequences

Part	Description
A	• Defining a sequence (nth term formula or recurrent relation) • Finding the nth term of a sequence • Finding the common ratio of a sequence
B	• Sum of the first n terms of a sequence • Sequences of non-numerical terms • Systems of equations containing sequence terms
C	• Word problems • Combinations of arithmetic and geometric sequences

Limit of a Sequence

Part	Description
A	• Evaluation of limits of sequences containing polynomials and rational expressions • Limit laws – sum of limits, difference of limits, product of limits and ratio of limits laws
B	• Evaluation of limits with trigonometric, exponential and logarithmic functions
C	• Use of the limit of the sequence (1+1/n)^n • Evaluation of limits with radicals • Evaluation of limits containing sums of sequences

Infinite Series

Part	Description
A	• Summation notation • Finding of the first term and of the common ratio of a geometric sequence • The sum of an infinite geometric series
B	• Periodic numbers (repeating decimals) • Finding of all x for which a series diverges or converges • Solving equations with infinite series • Word problems

Differential and Integral Calculus

Limits and Continuity

Part	Description
A	• Calculating limits – polynomials and rational functions • One-sided limits • Finding limits of functions from graphs
B	• Calculating limits – trigonometric functions • Calculating limits – functions with radicals • Continuity, discontinuity points
C	• Theoretical aspects related to limits calculations

Derivative

Part	Description
A	• Geometric interpretation of the derivative • Derivatives of elementary functions
B	• Derivative of a product of functions • Derivative of a quotient function • Derivative of a composite function
C	• Derivative of a composite function – complex problems

Applications of Derivatives

Part	Description
A	• Higher order derivatives • Function's monotonicity • Local extrema
B	• Concavity and convexity of a function • Global extrema
C	• Tangent line to graph of a function • Normal line to graph of a function • Asymptotes of a graph of a function • Calculating limits using L'Hospital's rule • Word problems, problems with parameter

Primitive Function

Part	Description
A	• Geometric interpretation of the antiderivative (primitive function) • Solving simple indefinite integrals (Finding a primitive function)
B	• Solving integrals requiring simplification of expressions • Solving integrals by substitution • Solving integrals by Parts
C	• Integrals solved by substitution – complex problems • Integrals solved by Parts – complex problems • Solving integrals requiring partial fraction decomposition • Integrals with parameters

Definite Integral

Part	Description
A	• Evaluation of simple definite integrals
B	• Evaluating integrals requiring simplification of expressions • Evaluating integrals using substitution • Evaluating integrals by Parts
C	• Evaluating integrals using substitution – complex problems • Evaluating integrals by Parts – complex problems • Evaluating integrals requiring partial fraction decomposition • Problems with parameters

Applications of Definite Integral

Part	Description
A	• The area of a plane region
B	• The volume of a solid
C	• The area of a plane region – complex problems • The volume of a solid – complex problems • Applications to physics

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