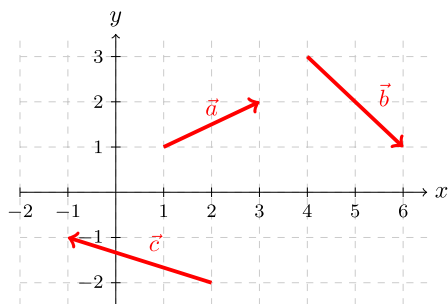


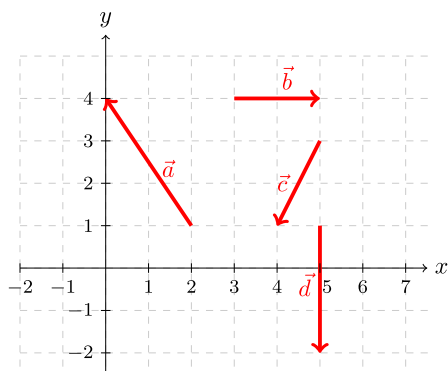
## Points and vectors - level B

1. Given vectors  $\vec{a}$ ,  $\vec{b}$ ,  $\vec{c}$ , find  $2\vec{a} + 3\vec{b} - \vec{c}$ .



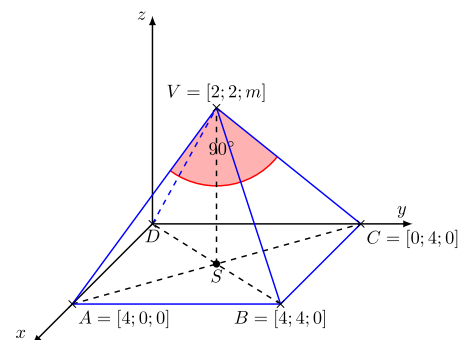
- (a) (7; 7)
- (b) (7; 0)
- (c) (7; -5)
- (d) (13; -5)

2. Given vectors  $\vec{a}$ ,  $\vec{b}$ ,  $\vec{c}$ ,  $\vec{d}$ , find  $\vec{a} + \vec{b} + \vec{c} + \vec{d}$ .



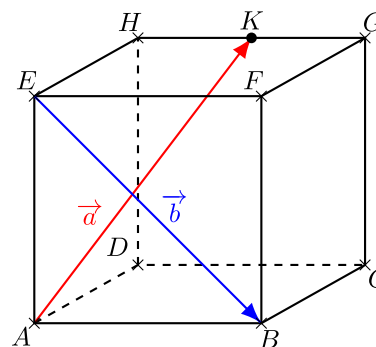
- (a) (2; -3)
- (b) (-1; -2)
- (c) (17; 7)
- (d) (6; 10)

3. Let  $ABCDV$  be a right pyramid with a square base, such that its opposite edges contain a right angle (see the picture). Specify the missing coordinate of the apex  $V$ .



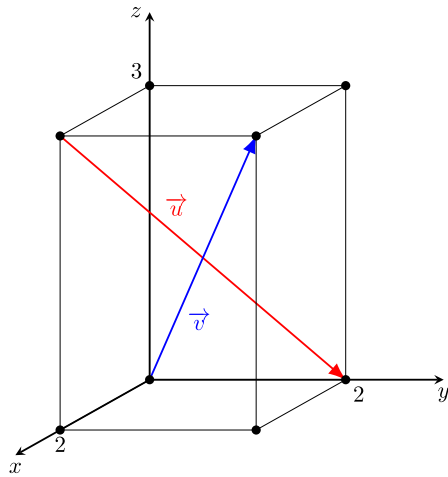
- (a)  $m = -2\sqrt{2}$
- (b)  $m = 4\sqrt{2}$
- (c)  $m = 2\sqrt{2}$
- (d)  $m = \sqrt{2}$

4. In the cube  $ABCDEFGH$  find the angle  $\varphi$  between the vectors  $\vec{b} = \vec{EB}$  and  $\vec{a} = \vec{AK}$ , where  $K$  is the midpoint of  $HG$ . Round  $\varphi$  to the nearest degree. Help: Choose the appropriate coordinate system.



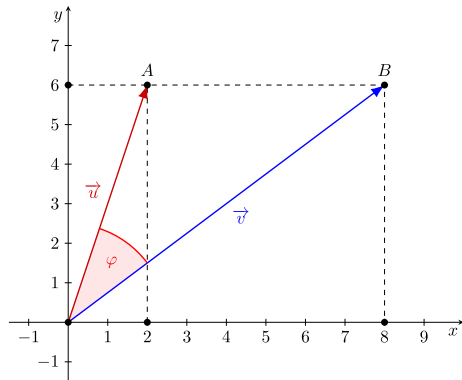
- (a)  $\varphi \doteq 80^\circ$
- (b)  $\varphi \doteq 76^\circ$
- (c)  $\varphi \doteq 104^\circ$
- (d)  $\varphi \doteq 100^\circ$

5. The vectors  $\vec{u}$  and  $\vec{v}$  are given by the figure. Find cosine of the angle  $\varphi$  between  $\vec{u}$  and  $\vec{v}$ . Help: Use the dot product of the given vectors.



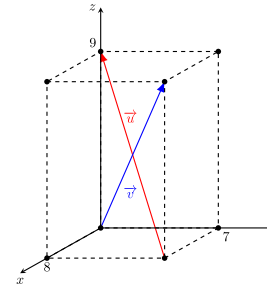
- (a)  $\cos \varphi = -\frac{9}{17}$   
 (b)  $\cos \varphi = -\frac{\sqrt{17}}{2\sqrt{13}}$   
 (c)  $\cos \varphi = \frac{9}{17}$   
 (d)  $\cos \varphi = \frac{\sqrt{17}}{2\sqrt{13}}$

6. The vectors  $\vec{u}$  and  $\vec{v}$  are given by the figure. Find cosine of the angle  $\varphi$  between  $\vec{u}$  and  $\vec{v}$ . Help: Use the dot product of the given vectors.



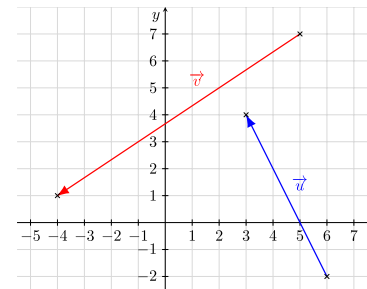
- (a)  $\cos \varphi = \frac{13\sqrt{10}}{50}$   
 (b)  $\cos \varphi = \frac{3\sqrt{10}}{10}$   
 (c)  $\cos \varphi = \frac{970}{50}$   
 (d)  $\cos \varphi = \frac{\sqrt{10}}{5}$

7. Find the coordinates of the vectors  $\vec{u}$  and  $\vec{v}$  given by the picture and evaluate their dot product.



- (a)  $\vec{u} = (8; 7; -9)$ ;  $\vec{v} = (-8; -7; -9)$ ;  $\vec{u} \cdot \vec{v} = (-64; -49; 81)$   
 (b)  $\vec{u} = (-8; -7; 9)$ ;  $\vec{v} = (8; 7; 9)$ ;  $\vec{u} \cdot \vec{v} = (-64; -49; 81)$   
 (c)  $\vec{u} = (-8; -7; 9)$ ;  $\vec{v} = (8; 7; 9)$ ;  $\vec{u} \cdot \vec{v} = -32$   
 (d)  $\vec{u} = (-8; -7; 9)$ ;  $\vec{v} = (8; 7; 9)$ ;  $\vec{u} \cdot \vec{v} = 0$

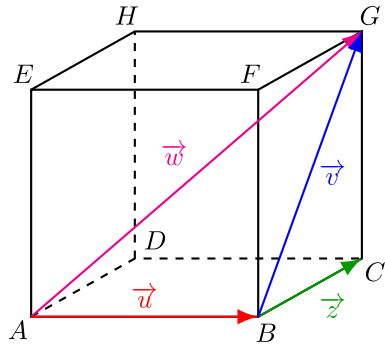
8. Find the coordinates of the vectors  $\vec{u}$  and  $\vec{v}$  given by the picture and evaluate their dot product.



- (a)  $\vec{u} = (3; -6)$ ;  $\vec{v} = (9; 6)$ ;  $\vec{u} \cdot \vec{v} = -9$   
 (b)  $\vec{u} = (-3; 6)$ ;  $\vec{v} = (-9; -6)$ ;  $\vec{u} \cdot \vec{v} = 9$   
 (c)  $\vec{u} = (-3; 6)$ ;  $\vec{v} = (-9; -6)$ ;  $\vec{u} \cdot \vec{v} = -9$   
 (d)  $\vec{u} = (3; -6)$ ;  $\vec{v} = (9; 6)$ ;  $\vec{u} \cdot \vec{v} = 0$

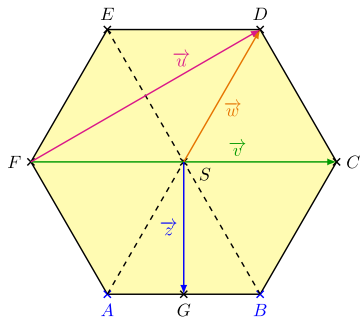
9. The vectors  $\vec{u}$ ,  $\vec{v}$ ,  $\vec{w}$ ,  $\vec{z}$  are indicated in a cube shown in the figure. The cube edge length is 1. Find the dot products of:

$$\vec{v} \cdot \vec{z}, \vec{u} \cdot \vec{v}, \vec{w} \cdot \vec{u}.$$



- (a)  $\vec{v} \cdot \vec{z} = 1, \vec{u} \cdot \vec{v} = 1, \vec{w} \cdot \vec{u} = \sqrt{3}$
- (b)  $\vec{v} \cdot \vec{z} = 1, \vec{u} \cdot \vec{v} = 0, \vec{w} \cdot \vec{u} = 1$
- (c)  $\vec{v} \cdot \vec{z} = \frac{\sqrt{2}}{2}, \vec{u} \cdot \vec{v} = 1, \vec{w} \cdot \vec{u} = \sqrt{3}$
- (d)  $\vec{v} \cdot \vec{z} = \sqrt{2}, \vec{u} \cdot \vec{v} = 0, \vec{w} \cdot \vec{u} = 1$

10. Let  $ABCDEF$  be a regular hexagon with the centre  $S$  and the side of length 3 cm. The point  $G$  is the midpoint of the segment  $AB$ . The vectors  $\vec{u}$ ,  $\vec{v}$ ,  $\vec{w}$ ,  $\vec{z}$  are indicated in the hexagon shown in the picture. Find the dot product of:  $\vec{v} \cdot \vec{w}$ ,  $\vec{v} \cdot \vec{z}$  and  $\vec{v} \cdot \vec{u}$ .



- (a)  $\vec{v} \cdot \vec{w} = 9, \vec{v} \cdot \vec{z} = 0, \vec{v} \cdot \vec{u} = 9\sqrt{6}$
- (b)  $\vec{v} \cdot \vec{w} = \frac{9}{2}, \vec{v} \cdot \vec{z} = 0, \vec{v} \cdot \vec{u} = 9\sqrt{6}$
- (c)  $\vec{v} \cdot \vec{w} = \frac{9}{2}, \vec{v} \cdot \vec{z} = 1, \vec{v} \cdot \vec{u} = 27$
- (d)  $\vec{v} \cdot \vec{w} = 9, \vec{v} \cdot \vec{z} = 0, \vec{v} \cdot \vec{u} = 27$

Answers (Points and vectors - level B): 1d, 2b, 3c, 4c, 5a, 6a, 7c, 8c, 9b, 10d,