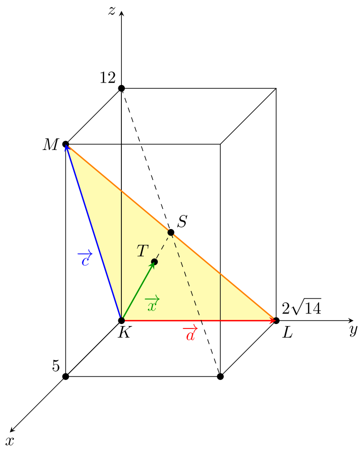
Points and vectors - level A

1. Let there be a triangle KLM and vectors $\overrightarrow{a}$, $\overrightarrow{c}$ in the coordinate system. Triangle KLM and vectors $\overrightarrow{a}$, $\overrightarrow{c}$ are given in the coordinate system shown in the picture. Point T is the centroid of the triangle KLM. Express vector $\overrightarrow{x}$, where $\overrightarrow{x} = \overrightarrow{KT}$ as a linear combination of $\overrightarrow{a}$ and $\overrightarrow{c}$ and evaluate $|\overrightarrow{x}|$.



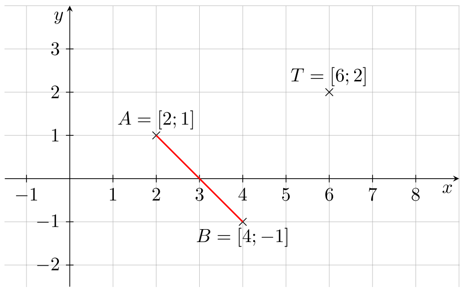
(a) $\overrightarrow{x} = \frac{1}{4}\overrightarrow{a} + \frac{1}{4}\overrightarrow{c}$, $|\overrightarrow{x}| = \frac{225}{12}$

(b) $\overrightarrow{x} = \frac{2}{3}\overrightarrow{a} + \frac{2}{3}\overrightarrow{c}$, $|\overrightarrow{x}| = 10$

(c) $\overrightarrow{x} = \frac{1}{2}\overrightarrow{a} + \frac{1}{2}\overrightarrow{c}$, $|\overrightarrow{x}| = \frac{15}{2}$

(d) $\overrightarrow{x} = \frac{1}{3}\overrightarrow{a} + \frac{1}{3}\overrightarrow{c}$, $|\overrightarrow{x}| = 5$

2. We are given points $A = [2;1]$, $B = [4;-1]$, and $T = [6;2]$, where point T is the centroid of triangle ABC. Find the coordinates of C, which is the vertex of ABC.



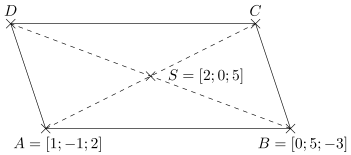
(a) $C = [8;5]$

(b) $C = [8;4]$

(c) $C = [9;6]$

(d) $C = [12;6]$

3. We are given points $A = [1;-1;2]$, $B = [0;5;-3]$, $S = [2;0;5]$. Point S is the centre of a parallelogram $ABCD$. Find the coordinates of vertices C and D.



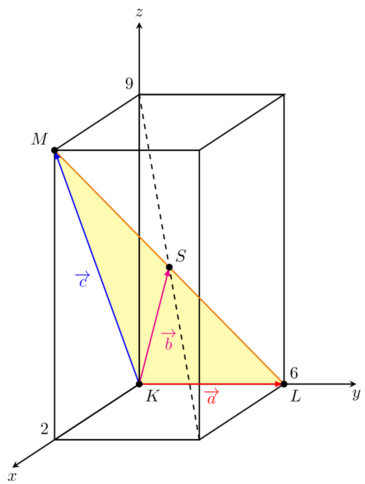
(a) $C = [1;1;3]; D = [2;-5;8]$

(b) $C = [-3;-1;-8]; D = [-4;5;-13]$

(c) $C = [3;1;8]; D = [4;-5;13]$

(d) $C = [4;-5;13]; D = [3;1;8]$

4. The picture shows the triangle KLM with indicated vectors $\overrightarrow{a}$, $\overrightarrow{b}$, $\overrightarrow{c}$ in a coordinate system. What are the vector coordinates $\overrightarrow{b}$? Express $\overrightarrow{b}$ as a linear combination of $\overrightarrow{a}$ and $\overrightarrow{c}$.



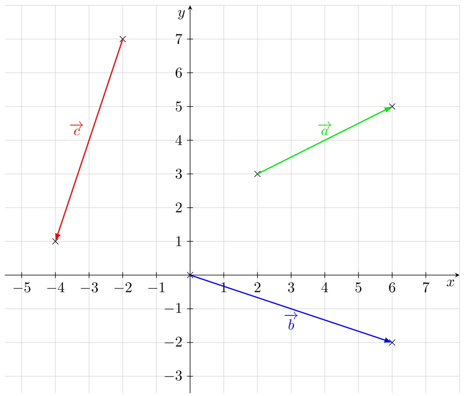
(a) $\overrightarrow{b} = (1;3;4{,}5)$; $\overrightarrow{b} = \frac{1}{2}\overrightarrow{a} + \frac{1}{2}\overrightarrow{c}$

(b) $\overrightarrow{b} = (3;1;4{,}5)$; $\overrightarrow{b} = \frac{1}{2}\overrightarrow{a} + \frac{1}{2}\overrightarrow{c}$

(c) $\overrightarrow{b} = (1;3;4{,}5)$; $\overrightarrow{b} = \overrightarrow{a} + \overrightarrow{c}$

(d) $\overrightarrow{b} = (3;1;4{,}5)$; $\overrightarrow{b} = \overrightarrow{a} + \overrightarrow{c}$

5. Given the vectors $\overrightarrow{a}$, $\overrightarrow{b}$, $\overrightarrow{c}$ shown in the picture, express a vector $\overrightarrow{b}$ as a linear combination of vectors $\overrightarrow{a}$ and $\overrightarrow{c}$.



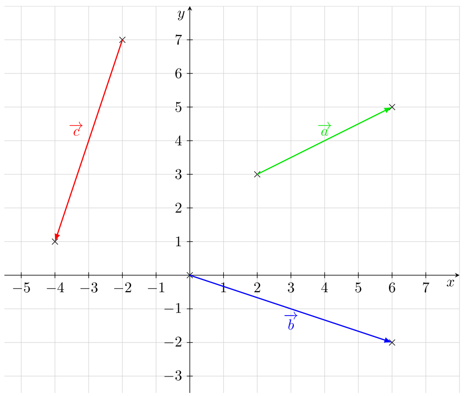
(a) $\overrightarrow{b} = 2\overrightarrow{a} - \overrightarrow{c}$

(b) $\overrightarrow{b} = -2\overrightarrow{a} - \overrightarrow{c}$

(c) $\overrightarrow{b} = 2\overrightarrow{a} + \overrightarrow{c}$

(d) $\overrightarrow{b} = -2\overrightarrow{a} + \overrightarrow{c}$

6. Given the vectors $\overrightarrow{a}$, $\overrightarrow{b}$, $\overrightarrow{c}$ shown in the picture, express a vector $\overrightarrow{c}$ as a linear combination of vectors $\overrightarrow{a}$ and $\overrightarrow{b}$.



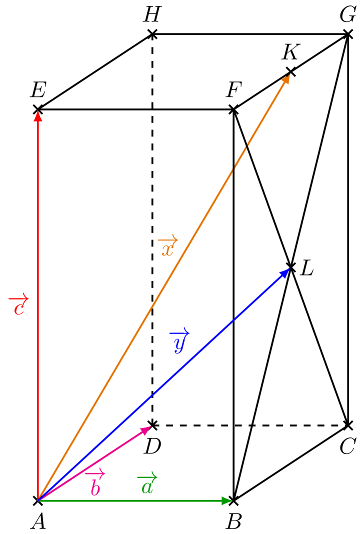
(a) $\overrightarrow{c} = -\frac{3}{2}\overrightarrow{a} + \overrightarrow{b}$

(b) $\overrightarrow{c} = -2\overrightarrow{a} + \overrightarrow{b}$

(c) $\overrightarrow{c} = -2\overrightarrow{a} + \frac{3}{2}\overrightarrow{b}$

(d) $\overrightarrow{c} = -\overrightarrow{a} + \frac{1}{2}\overrightarrow{b}$

7. The picture shows a rectangular cuboid $ABCDEFGH$ with $\overrightarrow{a} = \overrightarrow{AB}$, $\overrightarrow{b} = \overrightarrow{AD}$, $\overrightarrow{c} = \overrightarrow{AE}$, $\overrightarrow{x} = \overrightarrow{AK}$ and $\overrightarrow{y} = \overrightarrow{AL}$. Point K is the midpoint of FG and point L is the centre of face $BCGF$. Express vectors $\overrightarrow{x}$ and $\overrightarrow{y}$ as a linear combination of vectors $\overrightarrow{a}$, $\overrightarrow{b}$, $\overrightarrow{c}$.



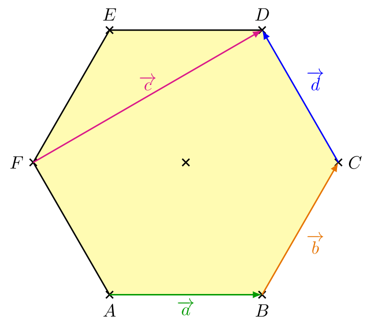
(a) $\overrightarrow{x} = \overrightarrow{a} + \frac{1}{2}\overrightarrow{b} + \frac{1}{2}\overrightarrow{c}$; $\overrightarrow{y} = \frac{1}{2}\overrightarrow{a} + \frac{1}{2}\overrightarrow{b} + \frac{1}{2}\overrightarrow{c}$

(b) $\overrightarrow{x} = \frac{1}{2}\overrightarrow{a} + \overrightarrow{b} + \frac{1}{2}\overrightarrow{c}$; $\overrightarrow{y} = \overrightarrow{a} - \frac{1}{2}\overrightarrow{b} + \frac{1}{2}\overrightarrow{c}$

(c) $\overrightarrow{x} = \overrightarrow{a} + \frac{1}{2}\overrightarrow{b} + \overrightarrow{c}$; $\overrightarrow{y} = \overrightarrow{a} + \frac{1}{2}\overrightarrow{b} + \frac{1}{2}\overrightarrow{c}$

(d) $\overrightarrow{x} = \overrightarrow{a} + \frac{1}{2}\overrightarrow{b} + \frac{1}{2}\overrightarrow{c}$; $\overrightarrow{y} = \overrightarrow{a} - \frac{1}{2}\overrightarrow{b} + \frac{1}{2}\overrightarrow{c}$

8. In a regular hexagon $ABCDEF$ shown in the picture, let $\overrightarrow{a} = \overrightarrow{AB}$, $\overrightarrow{b} = \overrightarrow{BC}$, $\overrightarrow{c} = \overrightarrow{FD}$ and $\overrightarrow{d} = \overrightarrow{CD}$. Express vectors $\overrightarrow{c}$ and $\overrightarrow{d}$ as a linear combination of vectors $\overrightarrow{a}$ and $\overrightarrow{b}$.



(a) $\overrightarrow{c} = \overrightarrow{a} + \overrightarrow{b}$; $\overrightarrow{d} = \overrightarrow{a} - \overrightarrow{b}$

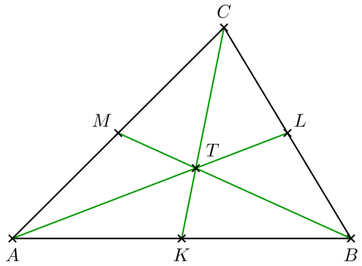
(b) $\overrightarrow{c} = \overrightarrow{a} + \overrightarrow{b}$; $\overrightarrow{d} = \overrightarrow{b} - \overrightarrow{a}$

(c) $\overrightarrow{c} = 2\overrightarrow{a} + \overrightarrow{b}$; $\overrightarrow{d} = \overrightarrow{b} - \overrightarrow{a}$

(d) $\overrightarrow{c} = 2\overrightarrow{a} + 2\overrightarrow{b}$; $\overrightarrow{d} = 2\overrightarrow{b} - 0{,}5\overrightarrow{a}$

9. In a triangle ABC, let K, L and M be the midpoints of AB, BC and AC consecutively and let T be the centroid of ABC. What are the values of coefficients k, l and m if

$$\overrightarrow{TM} = k \cdot \overrightarrow{BT};\ \overrightarrow{ML} = l \cdot \overrightarrow{BA};\ \overrightarrow{CK} = m \cdot \overrightarrow{TC}\ .$$



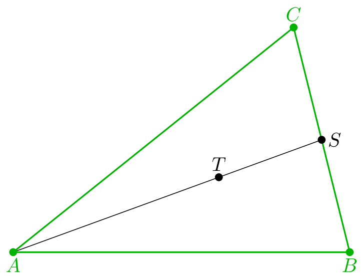
(a) $k = \frac{1}{2};\ l = \frac{1}{2};\ m = -\frac{3}{2}$

(b) $k = \frac{1}{2};\ l = -\frac{1}{2};\ m = \frac{3}{2}$

(c) $k = \frac{1}{2};\ l = -\frac{1}{2};\ m = -\frac{2}{3}$

(d) $k = \frac{1}{2};\ l = -\frac{1}{2};\ m = -\frac{3}{2}$

10. Let ABC be a triangle. In the picture below, the midpoint of the side BC and the centroid of the triangle are indicated. Out of the following vector relations select the one that is not true.



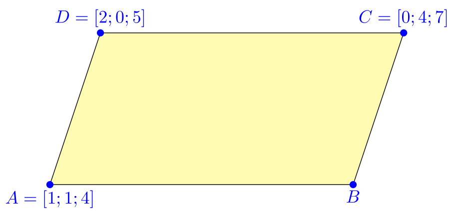
(a) $\overrightarrow{ST} = \frac{1}{2}\overrightarrow{AT}$

(b) $\overrightarrow{ST} = -\frac{1}{3}\overrightarrow{AS}$

(c) $\overrightarrow{AT} = \frac{2}{3}\overrightarrow{AS}$

(d) $\overrightarrow{SA} = -3\overrightarrow{TS}$

11. We are given the points $A = [1;1;4]$, $C = [0;4;7]$ and $D = [2;0;5]$ as seen in the picture below. What are the coordinates of a point B, if $ABCD$ is a parallelogram.



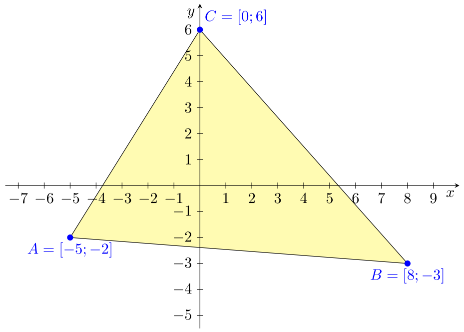
(a) $B = [-3;3;-2]$

(b) $B = [-1;5;6]$

(c) $B = [-2;4;3]$

(d) $B = [3;-3;2]$

12. Find the coordinates of the centroid of the triangle ABC. Triangle ABC is shown in the picture.



(a) $T = [1;1]$

(b) $T = \left[1;\frac{1}{3}\right]$

(c) $T = \left[\frac{4}{3};0\right]$

(d) $T = [1;0]$

Answers (Points and vectors - level A): 1d, 2d, 3c, 4a, 5c, 6b, 7c, 8b, 9d, 10a, 11b, 12b,