Scheduling a Sport Tournament with Methods of Discrete Mathematics

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joint work with

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Summer Mathematics Camp - Math4U
June 11th, 2018

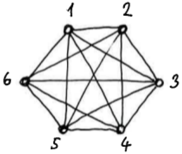
A Tournament

How to assemble a schedule?

Find a schedule of a tournament of 6 teams.

A Tournament

Graph Model



Complete graph decomposition into 1-factors

K_6

A Tournament

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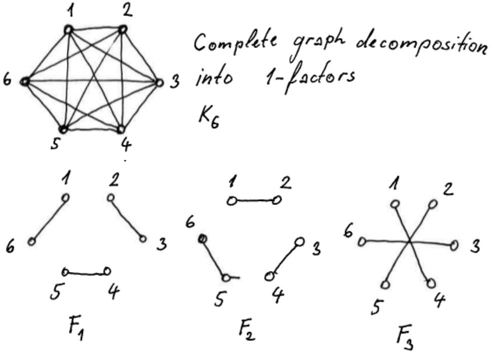

A Tournament

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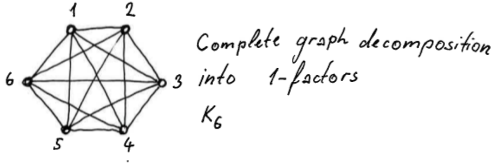

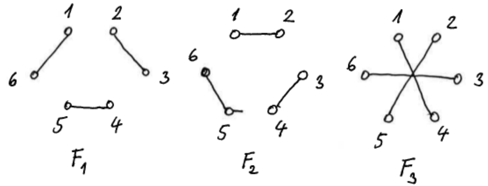

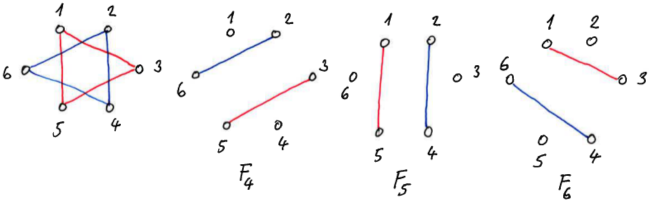

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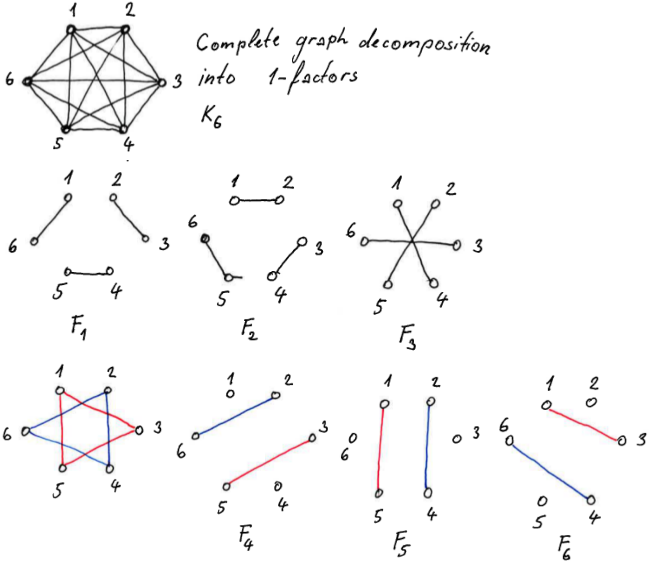

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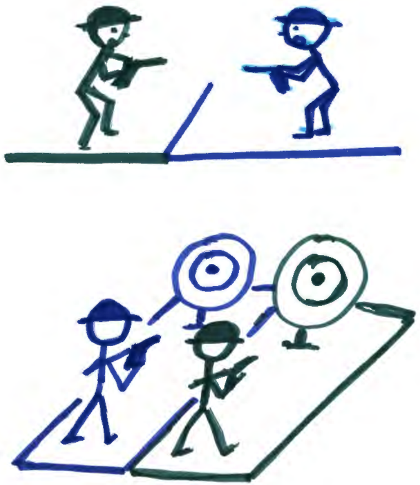
3. players evenly swap the shooting tracks
4. lengths of pauses are as even as possible for each player



Notation and basic calculations

We denote: N – number of players registered
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- for even number of players $N = 2n$
  - number of matches $Z = \binom{2n}{2} = \frac{2n\cdot(2n-1)}{2} = n \cdot (2n-1)$
  - number of matches in one round n
  - number of rounds $2n - 1$

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- for odd number of players $N = 2n-1$
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  - number of matches in one round $n-1$
  - number of rounds $2n-1$

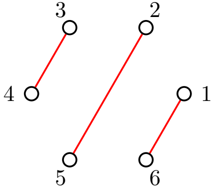
Tournament as a decomposition of complete graph

Example for $N = 6$

For 6 players number of matches is $Z = \binom{6}{2} = 15$.

In one round 3 matches.

Number of rounds 5.



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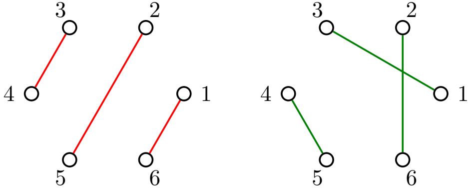

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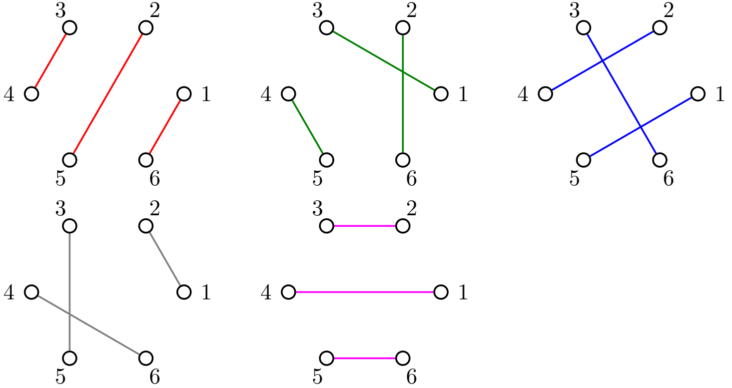

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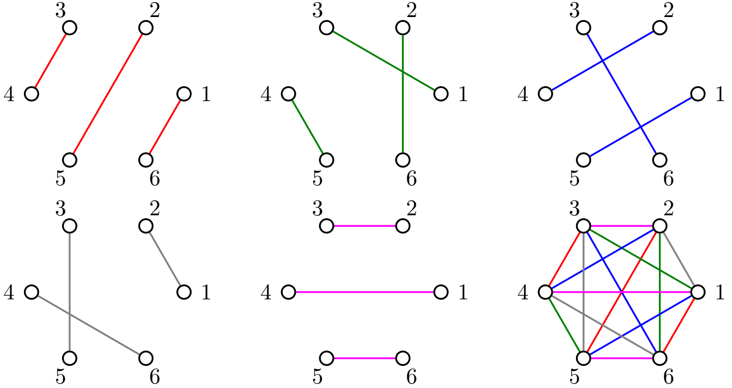

1-factorization of K_6

Canonical factorization

Definition

Denote F_1,F_2,.....,F_{2n-1} 1-factors of K_{2n}.
Factorization is called canonical if
$F_i = \{\{2n, i\}\} \cup \{\{i + k, i - k\}; k = 1, 2, ..., n - 1, i = 1, ..., 2n - 1\}$,
where $i + k$ and $i - k$ are taken $\pmod{2n - 1}$.

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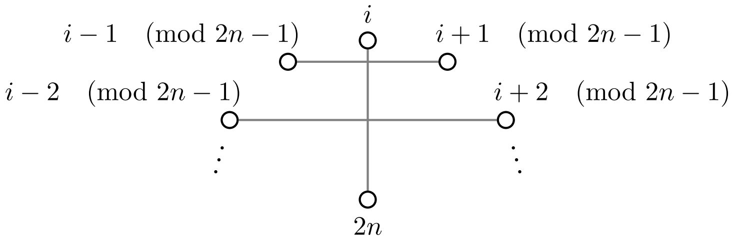

Factor F_i of K_{2n}

Schedule from canonical factorization

for 6 players

	1	2	3
F_1	61	25	34
F_2	62	31	45
F_3	63	24	15
F_4	64	35	21
F_5	65	14	23

Schedule from canonical factorization

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for 5 players

	0	1	2
F_1	1	25	34
F_2	2	31	45
F_3	3	42	51
F_4	4	35	12
F_5	5	14	32

Schedule from canonical factorization

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How to ensure switching the sides??

Home - Away Pattern

- soccer tournament - match placement – Home or Away ??

Home - Away Pattern

- soccer tournament - match placement – Home or Away ??
- edge orientation

i,k	1	2	3
F_1	$\overleftarrow{61}$	$\overrightarrow{25}$	$\overleftarrow{34}$
F_2	$\overrightarrow{62}$	$\overrightarrow{31}$	$\overleftarrow{45}$
F_3	$\overleftarrow{63}$	$\overrightarrow{42}$	$\overleftarrow{51}$
F_4	$\overrightarrow{64}$	$\overrightarrow{53}$	$\overleftarrow{12}$
F_5	$\overleftarrow{65}$	$\overrightarrow{14}$	$\overleftarrow{23}$

Home - Away Pattern

- soccer tournament - match placement – Home or Away ??
- edge orientation
- breaks, number of breaks

i,k	1	2	3
F_1	$\overleftarrow{61}$	$\overrightarrow{25}$	$\overleftarrow{34}$
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F_4	$\overrightarrow{64}$	$\overrightarrow{53}$	$\overleftarrow{12}$
F_5	$\overleftarrow{65}$	$\overrightarrow{14}$	$\overleftarrow{23}$

player,round	1	2	3	4	5
1	A	H	A	H	A
2	A	H	$\underline{H}$	A	H
3	H	A	$\underline{A}$	H	A
4	A	H	A	H	$\underline{H}$
5	H	A	H	A	$\underline{A}$
6	H	A	H	A	H

Home - Away Pattern

- soccer tournament - match placement – Home or Away ??
- edge orientation
- breaks, number of breaks
- complementarity property

i,k	1	2	3
F_1	$\overleftarrow{61}$	$\overrightarrow{25}$	$\overleftarrow{34}$
F_2	$\overrightarrow{62}$	$\overrightarrow{31}$	$\overleftarrow{45}$
F_3	$\overleftarrow{63}$	$\overrightarrow{42}$	$\overleftarrow{51}$
F_4	$\overrightarrow{64}$	$\overrightarrow{53}$	$\overleftarrow{12}$
F_5	$\overleftarrow{65}$	$\overrightarrow{14}$	$\overleftarrow{23}$

player,round	1	2	3	4	5
1	A	H	A	H	A
2	A	H	$\underline{H}$	A	H
3	H	A	$\underline{A}$	H	A
4	A	H	A	H	$\underline{H}$
5	H	A	H	A	$\underline{A}$
6	H	A	H	A	H

Home - Away Pattern

- soccer tournament - match placement – Home or Away ??
- edge orientation
- breaks, number of breaks
- complementarity property
- side at the shooting track – left (L), right (R)

i,k	1	2	3
F_1	16	25	43
F_2	62	54	31
F_3	36	15	42
F_4	64	53	21
F_5	56	14	32

player,round	1	2	3	4	5
1	L	R	L	R	L
2	L	R	$\underline{R}$	L	R
3	R	L	$\underline{L}$	R	L
4	L	R	L	R	$\underline{R}$
5	R	L	R	L	$\underline{L}$
6	R	L	R	L	R

Home - Away Pattern

For odd number of players, Home-Away pattern without breaks exists.
Example for $N = 5$.

	0	1	2
F_1	1	25	34
F_2	2	45	31
F_3	3	51	42
F_4	4	53	12
F_5	5	14	23

player,round	1	2	3	4	5
1	-	L	R	L	R
2	R	-	L	R	L
3	L	R	-	L	R
4	R	L	R	-	L
5	L	R	L	R	-

Scheduling with minimum number of breaks

Theorem (Werra, 1967)

For $n \geq 1$, there exists a compact schedule for K_{2n} into $2n-1$ rounds with the minimum number $2n-2$ of breaks and a compact schedule for K_{2n-1} into $2n-1$ rounds without breaks.

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Orientation of edges for the minimum number of breaks in canonical factorization.

1. For each $i \in \{1, 2, \ldots, 2n-1\}$ the edge $\{2n, i\}$ is oriented as $(i, 2n)$ if i is odd, or as $(2n, i)$ if i is even.
2. For each $i \in \{1, 2, \ldots, 2n-1\}$ the edge $\{i+k, i-k\}$ is oriented as $(i+k, i-k)$ if k is odd, or as $(i-k, i+k)$ if k is even.

How to ensure an even swapping of shooting tracks for each player?

Latin square

Latin square L of order n is an $n \times n$ table filled with n distinct symbols, so that in each row and in each column each symbol appears exactly once. **Euler,1782**

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♡	♣	□
♣	□	♡
□	♡	♣

1	4	2	3
2	3	1	4
4	1	3	2
3	2	4	1

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♡	♣	□
♣	□	♡
□	♡	♣

1	4	2	3
2	3	1	4
4	1	3	2
3	2	4	1

L_+ of order 5

+	1	2	3	4	5
1	2	3	4	5	1
2	3	4	5	1	2
3	4	5	1	2	3
4	5	1	2	3	4
5	1	2	3	4	5

Latin Squares

Isomorfismus

Let L and $\widehat{L}$ be Latin squares of order n, and let by E_1 denote the set of row indexes, by E_2 the set of column indexes, and by E_3, $\widehat{E_3}$ the symbol sets. If there exist bijections $\phi_1{:}E_1 \to E_1$, $\phi_2{:}E_2 \to E_2$, $\phi_3{:}E_3 \to \widehat{E_3}$, such that $\phi_3 L(i,j) = \widehat{L}(\phi_1(\mathrm{i}), \phi_2(\mathrm{j}))$ for each $i \in E_1$, $j \in E_2$ and $L(i,j) \in E_3$, then Latin squares L and $\widehat{L}$ are isomorphic.

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$L =$

2	3	1
3	1	2
1	2	3

$\to$

$\widehat{L} =$

1	2	3
2	3	1
3	1	2

Isomorphisms: $\phi_1 = \phi_2 = \mathrm{id}$, $\phi_3(2) = 1$, $\phi_3(1) = 3$, $\phi_3(3) = 2$

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$L =$

2	3	1
3	1	2
1	2	3

$\rightarrow$

$\widehat{L} =$

1	2	3
2	3	1
3	1	2

Isomorphisms: $\phi_1=\phi_2=$id, $\phi_3(2) = 1$, $\phi_3(1) = 3$, $\phi_3(3) =\ 2$

$$\phi_3 : \quad \begin{pmatrix} 2 & 1 & 3 \\ 1 & 2 & 3 \end{pmatrix}$$

Latin Squares

Transversal

A transversal of a $n \times n$ Latin square is a set of n cells, such that from each row and each column there is exactly one, and all n symbols are contained.

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2	3	1
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1	2	3

2	3	4	5	1
3	4	5	1	2
4	5	1	2	3
5	1	2	3	4
1	2	3	4	5

Latin Squares

Idempotent LS

Latin square of order n is idempotent if $L(i,i) = i$ for each $i \in \{1, ..., n\}$.

1	3	2
3	2	1
2	1	3

1	4	2	5	3
4	2	5	3	1
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3	1	4	2	5

Lemma

For n odd there exists an idempotent LS isomorphic to L_+ of order n.

+	1	2	3	4	5
1	2	3	4	5	1
2	3	4	5	1	2
3	4	5	1	2	3
4	5	1	2	3	4
5	1	2	3	4	5

$$\begin{pmatrix} 2 & 4 & 1 & 3 & 5 \\ 1 & 2 & 3 & 4 & 5 \end{pmatrix}$$

	1	2	3	4	5
1	1	4	2	5	3
2	4	2	5	3	1
3	2	5	3	1	4
4	5	3	1	4	2
5	3	1	4	2	5

Factorization of K_6 given by a Latin square

+	1	2	3	4	5
1	2	3	4	5	1
2	3	4	5	1	2
3	4	5	1	2	3
4	5	1	2	3	4
5	1	2	3	4	5

$$
\begin{aligned}
F_1 &= \{\{5,1\}, \{4,2\}, \{3,6\}\} \\
F_2 &= \{\{5,2\}, \{4,3\}, \{1,6\}\} \\
F_3 &= \{\{2,1\}, \{5,3\}, \{4,6\}\} \\
F_4 &= \{\{3,1\}, \{5,4\}, \{2,6\}\} \\
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\end{aligned}
$$

Factorization of K_6 given by a Latin square

+	1	2	3	4	5
1	2	3	4	5	1
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3	4	5	1	2	3
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Canonical factorization of K_6

$F'_i = \{\{2n, i\}\} \cup \{\{i+k, i-k\}\}$, $k = 1, 2, \ldots, n-1$,
$i = 1, \ldots, 2n-1$

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$$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 3 & 1 & 4 & 2 & 5 \end{pmatrix}$$

Latin squares and properties of latin squares used to assemble the schedule

- L_+ : A Latin square, which corresponds to an operation table of a finite additive group $(Z_n, +)$.

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Latin squares and properties of latin squares used to assemble the schedule

- L_+ : A Latin square, which corresponds to an operation table of a finite additive group $(Z_n, +)$.
- A Latin square L_+ of order n, where n is odd, has exactly n disjunctive transversals.
- For n odd, there exists idempotent Latin square of order n isomorphic to L_+.

Assembling the schedule

Example: 7 players and 3 shooting tracks

- Take an idempotent LS L_{id} isomorphic to L_+ of order 7, where the symbol $L(i,j)$ corresponds to the round in which the match between players i and j is played.

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	1	2	3	4	5	6	7
1	1	5	2	6	3	7	4
2	5	2	6	3	7	4	1
3	2	6	3	7	4	1	5
4	6	3	7	4	1	5	2
5	3	7	4	1	5	2	6
6	7	4	1	5	2	6	3
7	4	1	5	2	6	3	7

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4	6	3	7	4	1	5	2
5	3	7	4	1	5	2	6
6	7	4	1	5	2	6	3
7	4	1	5	2	6	3	7

kolo	1.stř	2.stř	3.stř
1.	27	45	63
2.	31	56	74
3.	42	67	15
4.	53	71	26
5.	64	12	37
6.	75	23	41
7.	16	34	52

Assembling the schedule

Example: 7 players and 3 shooting tracks

- Add an orientation in accordance with the canonical factorization.

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ko\hr	1	2	3	4	5	6	7
1	-	L	R	L	R	L	R
2	R	-	L	R	L	R	L
3	L	R	-	L	R	L	R
4	R	L	R	-	L	R	L
5	L	R	L	R	-	L	R
6	R	L	R	L	R	-	L
7	L	R	L	R	L	R	-

Summary of results

Claim

For the number of players $N \equiv 1 \pmod{2s}$ and the number of shooting tracks $s = 2, 3, 4$ there exists a schedulle, that fully satisfies conditions $1, 2, 3$ of the Harmonic Schedule and is near to satisfy the condition 4.

Summary of results

Claim

For the number of players $N \equiv 1 \pmod{2s}$ and the number of shooting tracks $s = 2, 3, 4$ there exists a schedulle, that fully satisfies conditions $1, 2, 3$ of the Harmonic Schedule and is near to satisfy the condition 4.

Note

If $k = p \cdot s$, where k is the number of rounds and s is the number of shooting tracks, then p is the number of sub-rounds.
The lengths of pauses for a player are:
– always except for the round in which the player does not play
0 **up to** $2p - 2$,
– once for the tournament for the round in which the player does not play
p **up to** $3p - 2$.

Example

$N = 9$, number of rounds $m = 9$, matches in one round $k = 4$.

Schedule given by L_{id} isomorphic to L_{+} using four transversals.

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kolo	1.	2.	3.	4.	5.	6.	7.	8.	9.
1.stř	65	76	87	98	19	21	32	43	54
2.stř	29	31	42	53	64	75	86	97	18
3.stř	47	58	69	71	82	93	14	25	36
4.stř	83	94	15	26	37	48	59	61	72

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4.stř	83	94	15	26	37	48	59	61	72

Odd Balanced Tournament Design.

$OBTD(n)$ on a $2n+1$-set V is an arrangement of all $\binom{2n+1}{2}$ distinct unordered pairs into an $n \times (2n+1)$ array such that

- each column of the array contains $2n$ distinct elements of V, and
- each element of V appears exactly twice in each row.

Balanced Tournament Design.

$BTD(n)$ on the set V of the size $2n$ is an ordering of all $\binom{2n}{2}$ distinct pairs into an $n \times 2n - 1$ array such that

- every element of V appears in each column exactly once
- every element of V appears in each row at most twice.

Theorem, 1974, Lamken, Vanstone

$BTD(n)$ exists if and only if n is natural number $n \neq 2$.

Thank you for your attention.