Monte Carlo methods

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Katedra aplikované matematiky
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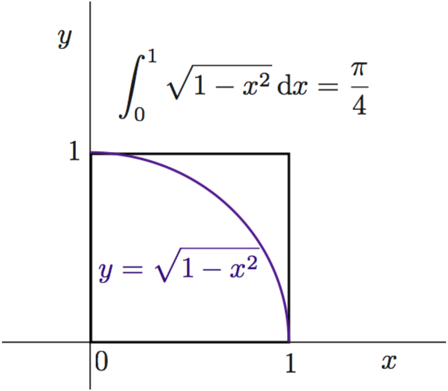
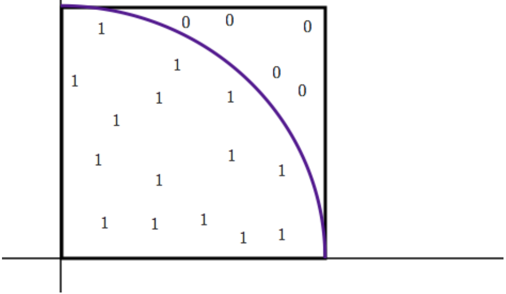
Area volume



How to solve it

$$x = 15 \qquad n = 20$$

$$\pi \doteq \frac{4x}{n} = \frac{4 \cdot 15}{20} = 3$$



Program

```
> with(Statistics): #Number of points in the unitary square [0,1]x[0,1] :
> CountSample:=10000:
> #Generating an appropriate sample of points
> X := RandomVariable(Uniform(0, 1)): Y := RandomVariable(Uniform(0, 1)):
> GunX:=Sample(X,CountSample): GunY:=Sample(Y,CountSample):
> #Compute the count of hits inside the part of the circle
> CountHit:=0:
> for i from 1 to CountSample do
> if (GunX[i])^2+(GunY[i])^2<1 then #x^2+y^2<1
      CountHit:=CountHit+1 end if
end do:
> #Print the result as a relative frequency
> print(`Aproximation of the number Pi:`); print(4.0*CountHit/CountSample);
Aproximation of the number Pi:3.132800000
```

Monty Hall Problem

http://www.mathwarehouse.com/monty-hall-simulation-online/

Program

```
> with(RandomTools):
> #CD...CountOfDoors, OD...OpenDoors, N...number of
simulation
> CD := 3: OD := 1: N := 1000:
> Success := 0:
```

Program

```
> #Main simulation loop
> for k from 1 to N do ;
> #Car...position of the car, Tip...assumed position before change,
> Car := Generate(integer(range = 1 .. CD));
> Tip := Generate(integer(range = 1 .. CD));
> #Construction of the initial situation
> Doors := array(1 .. CD);
> for i to CD do Doors[i] := 0 end do;
> Doors[Car] := 1; Doors[Tip] := 1.23456;
```

Program

```
> #Setting the open doors
> CountOD := 0;
> for i while CountOD< OD do
Index := Generate(integer(range = 1 .. > CD));
> if `and`(`and`(Doors[Index] < 3, Index <> Car), Index <> Tip)
then Doors[Index] := 3; CountOD := CountOD+1 end if
> end do; CountOD;
```

Program

```
> #Simulation when Tip is changed
> TestTip:=0 :
> for i from 1 while (TestTip<1) do
> Index:=Generate(integer(range=1..CD)):
> if ((Doors[Index]<2) and (Index <>Tip) and (Index=Car)) then
> TestTip:=1: Success:=1+ Success end if:
> if ((Doors[Index]<2) and (Index <>Tip)and(Index<>Car))then
> TestTip:=1 end if :
> end do:
> end do: Success:
```

Program

```
> print('Relative frequency, changed tip:');
> print(evalf(Success/N));
> print('Probability of the original tip:');
> print(1.0/CD); > Success:=0:
>
Relative frequency, changed tip:
0.6870000000
Probability of the original tip:
0.3333333333
```

Chebyshev's inequality for $Z = \frac{X}{n} = \frac{\text{CountHit}}{\text{CountSample}}$

$$P\left(\left|\frac{X}{n} - p\right| \geq \epsilon\right) \leq \frac{p \cdot (1-p)}{n \cdot \epsilon^2}$$

$$P\left(\left|\frac{X}{n} - p\right| \geq \epsilon\right) \leq \frac{1}{4 \cdot n \cdot \epsilon^2}$$

Expected value

Expected value of random variable V:

$$E\left(V\right)=\sum_{j=0}^{n}v_j\cdot P\left(V=v_j\right)$$

Properties:

$$E\left(U+V\right)=E\left(U\right)+E\left(V\right),\qquad E\left(\alpha\cdot U\right)=\alpha E\left(U\right)$$

Dispersion

Dispersion of random variable V:

$$D(V) = E\left[(V - E(V))^2\right] = \sum_{j=0}^{n} [v_j - E(V)]^2 \cdot P(V = v_j)$$

Properties:

$$D(\alpha \cdot U) = \alpha^2 D(U)$$

The case of the independence:

$$D(U + V) = D(U) + D(V)$$

Alternative random variable

Alternative random variable

$$Y \leftrightarrow A(p): \qquad P(Y=1)=p, \qquad P(Y=0)=(1-p)$$

$$E(Y)=p, \qquad D(Y)=p\cdot(1-p)$$

Binomial random variable

$$X = \sum_{j=1}^{n} Y_j \qquad Y_j \leftrightarrow A(p) \text{ INDEPENDENT}$$

Binomial random variable

$$X \leftrightarrow Bin(n, p) \qquad P(X = k) = \binom{n}{k} \cdot p^k \cdot (1-p)^{n-k}$$

$$E(X) = np, \qquad D(X) = np \cdot (1-p)$$

Average successness

$$Z = \frac{X}{n} \qquad X \leftrightarrow Bin(n, p)$$

$$E(Z) = p, \qquad D(Z) = \frac{p \cdot (1 - p)}{n}$$

Chebyshev's inequality

$$P\left(\left|V - E\left(V\right)\right| \geq \epsilon\right) \leq \frac{D\left(V\right)}{\epsilon^2}$$